Red Rose Senior Secondary School

Class: XII

Subject:MATHS

Chapter : 1 (Relations and Function)

1. Show that the relation R defined by $(a, b)\ R(c, d)\ \rightarrow a + d\ = b + c$ on the set $N\ \times\ N$ is an equivalence relation. ***(4) [2008]***
2. Let * be a binary operation on N given by $a * b\ =\ HCF\ (a, b)\ \ a, b\ \in\ N$. Write the value of 22 * 4. ***(1) [2009]***
3. Let $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$ Find whether the function f is bijective. ***(4) [2009]***
4. If f : R → R be defined by $f(x) = (3 - x^3)^{1/3}$, then find of $fof(x)$. ***(1)[2010]***
5. Show that the relation S defined on the set $N \times N$ by (a, b) S (c, d) ⇒a + d = b + c, is an equivalence relation. ***(4) [2010]***
6. Show that the relation S in the set A=$\{x \in Z : 0 \leq x \leq 12\}$ given by S = $\{(a, b): a, b \in Z, |a - b|$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. ***(4) [2010]***
7. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not. ***(1) [2011]***
8. Let f: R→R be defined as f(x) = 10x +7. Find the function $g: R \rightarrow R$ such that $gof\ = fog\ = I_R$ ***(4) [2011]***
9. A binary operation * on the set (0, 1, 2, 3, 4, 5} is defined as:
$$a * b = \begin{cases} a + b, if\ a + b < 6 \\ a + b - 6,\ \ if\ a + b\ \geq 6 \end{cases}$$
show that zero is the identity for this operation and each element ‘a’ of the set is invertible with 6-a, being the inverse of ‘a’. ***(4) [2011]***

10. The binary operation $*: R\ x\ R \rightarrow R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$. **(1) [2012]**

11. Show that $f: N \rightarrow N$, given by $f(x) = \begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ is both one-one and onto. ***(4) [2012]***

12. Consider the binary operation $*: R \times R \rightarrow R\ and\ o: R \times R \rightarrow R$ Defined as $a * b = |a-b|\ and\ a\ o\ b = a\ for\ all\ a, b \in R$. Show that '*' is commutative but not associate, 'o' is associative but not commutative. ***(4) [2012]***

13. Consider $f: R+ \rightarrow (4, \infty)\ given\ by\ f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. ***(4) [2013]***

14. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. ***(1) [2014]***

15. If the function $f : R \rightarrow R$ be given by $f(x) = x^2 + 2\ and\ g : R \rightarrow R$ be given by $g(x) = \frac{x}{x-1}, x \neq 1$, find $fog\ and\ gof$ and hence find $fog(2)\ and\ gof(-3)$. ***(4) [2014]***

16. $Consider\ f: R_+ \rightarrow [-9, \infty]\ given\ by\ f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \langle \frac{\sqrt{54+5y}-3}{5} \rangle$ ***(6) [2015]***

17. A binary operation * is defined on the set $x = R - \{-1\}$ by

$$x * y = x + y + xy, \forall\ x, y \in X$$

check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X. Or

Show that the binary operation * on $A = R - \{-1\}$ defined as $a * b = a + b + ab\ for\ all\ a, b \in A$ is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible. ***(6) [2016]***

18. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{ (a, b) : |a - b| \; is \; divisible \; by \; 2\}$ is an equivalence relation. Write all the equivalence classes of R. ***(4) [2015 Comptt.]***

19. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative with respect to $*$ on A.
    i) Find the identity element in A.
    ii) Find the invertible element of A.

Or

Consider $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is the bijective. Find the inverse of f and hence find $f^{-1}(0) and \; x$ such that $f^{-1}(x) = 2$. **6[2017]**

20. Given a non-empty set X, consider the binary operation $*: P(X) \times P(X) \rightarrow P(X)$ given by $A * B = A \cap B, \forall A, B \in P(X)$, where $P(X)$ is the power set X. show that * is commutative and associative and X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation *.

Or

Let $f: R - \{-\frac{4}{3}\} \rightarrow R$ be a funcion defined as $\boldsymbol{f(x)} = \frac{\boldsymbol{4x}}{\boldsymbol{3x+4}}$. Show that f is a one-one function. Also check whether f is an onto function or not. Hence find $f^{-1} \; in \; (range \; of \; f) \rightarrow R - \left\{-\frac{4}{3}\right\}$. **6 [2017]**

21. If $a * b$ denotes the larger of 'a' and 'b' and if $aob = (a + b) + 3$, then write the value of $(5)o(10)$, where * and o are bnary operation. **1 [2018]**

22. Let $A = \{x \in Z: 0 \leq x \leq 12\}$. show that $R = \{(a, b): a, b \in A, |a - b| \; is \; divisible \; by \; 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also the equivalence class [2].

or

Show that the function $f: R \to R$ defined by $\boldsymbol{f(x) = \frac{x}{x^2+1}}$, $\forall\, x \in R$ is neither one-one nor onto. Also, if $g: R \to R$ is defined as $g(x) = 2x - 1$,find fog. **6 [2018]**

22. Examine whether the operation * defined on $\boldsymbol{R}$. the set of all real numbers, by $\boldsymbol{a * b = \sqrt{a^2 + b^2}}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

2 [2019].

23. Check whether the relation R defined on the set $A = \{1,2,3,4,5,6\}$ as $R = \{(a, b): b = a + 1\}$ reflexive, symmetric or transitive.

or

Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N: y = 4x + 3, for\ some\ x \in N\}$. Show that f is invertible. Find its inverse. **4 [2019].**

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